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ABSTRACT

This unit views applications of elementary game theory to international relations. It is noted that of all the significant world problems, the nuclear arms race has proved one of the most intractable. The main concern of the module is to investigate a possible solution to the arms race, based on extending the classic two-person game of Prisoner's Dilemma and allowing for sequences of moves. An analysis of the generated model is followed with considerations of possible extensions to both new games and different scenarios. The material is designed to help students: (1) understand some basic concepts in game theory; (2) apply concepts to arms race analysis and other two-person conflict situations; (3) derive consequences of extended play of different games and scenarios; (4) illustrate consequences graphically; (5) state policy implications of an analysis; and (6) justify normative judgments based on analysis. The module contains exercises, with solutions, and concludes with a listing of references. (MP)

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UNIT 311

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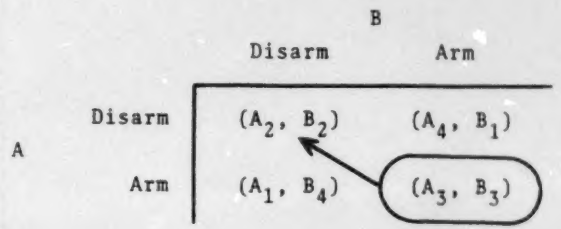
MODULES AND MONOGRAPHS IN UNDERGRADUATE MATHEMATICS AND ITS APPLICATIONS PROJECT

THE GEOMETRY OF THE ARMS RACE

by Steven J. Brams, Morton D. Davis,

and Philip D. Straffin Jr.



APPLICATIONS OF ELEMENTARY GAME THEORY TO INTERNATIONAL RELATIONS

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THE GEOMETRY OF THE ARMS RACE

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Intermodular Description Sheet: UMAP Unit 311

Title: THE GEOMETRY OF THE ARMS RACE

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Suggested Support Material:

References: See Section 9 of text.

Prerequisite Skills:
1. Be able to graph second-degree curves.
2. Be able to find maxima and minima of simple functions.
3. Knowledge of elementary probability concepts.

Output Skills:
1. Understand some basic concepts in game theory.
2. Apply these concepts to the analysis of arms races and other two-person conflict situations.
3. Derive consequences of extended play of different games and game scenarios.
4. Illustrate these consequences geometrically (graphically).
5. State policy implications of the analysis.
6. Justify normative judgments based on the analysis.

Other Related Units:
The Richardson Arms Race Model *(Unit 308)*

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MODULES AND MONOGRAPHS IN UNDERGRADUATE
MATHEMATICS AND ITS APPLICATIONS PROJECT (UMAP)

The goal of UMAP is to develop, through a community of users and developers, a system of instructional modules in undergraduate mathematics and its applications which may be used to supplement existing courses and from which complete courses may eventually be built.

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1. INTRODUCTION

Of all the significant problems that confront the world, the nuclear arms race between the United States and the Soviet Union has proved one of the most intractable. Its intractability, however, stems not from the awesome amounts both sides have expended on arms, nor even in the millions of lives at stake should the arms race culminate in a nuclear war. While these facts help to explain why the arms race looms so large in our lives, they do not explain why this race has proved so difficult to slow down.

Since the benefits and costs of the arms race to each nation are dependent on what *both* nations do, it is helpful to think of the arms race as a "game." A *game* is an interdependent decision situation in which the outcome depends not just on chance but on the actions that two or more players, or participants, take when they make choices in the game.

The simplest game-theoretic model which has been used to analyze the arms race is the two-person game of Prisoners' Dilemma, in which each player is assumed to have two strategies.[1] Of course, modeling the arms race by any model which assumes that the nations as players have only two strategies, leading to well-defined payoffs, is a drastic oversimplification. However, this particular simplified model has the advantage that it exhibits, in a strikingly simple way, an explanation of the fundamental intractability of the arms race based only on the consequences of rational behavior by the players.

Our main concern in this module is to investigate a possible solution to the arms race, based on extending the classic Prisoners' Dilemma game to allow for *scenarios*, or sequences of moves. We begin our analysis by reviewing, in Section 2, the Prisoners' Dilemma model of the arms race. In Sections 3, 4, and 5 we present a scenario of conditional cooperation and analyze when that scenario is advantageous to the players. In Section 6, we explore some policy implications of the model, and discuss further some of its limitations. In Section 7 we summarize our analysis and consider possible extensions of our framework to both new games and different game scenarios.

[1] Rapoport and Chammah (1965); for a recent review of the literature on Prisoners' Dilemma, see Brams (1976: chs. 4 and 8).

2. PRISONERS' DILEMMA AND THE ARMS RACE

Prisoners' Dilemma is a two-person game that is illustrated in Figure 1. We shall not describe the original story that gives Prisoners' Dilemma its name but shall instead interpret it in the context of the arms race between the superpowers, whom we call A and B.

		B: Disarm	B: Arm
A	Disarm	(A_2, B_2)	(A_4, B_1)
	Arm	(A_1, B_4)	(A_3, B_3)

Figure 1. The arms race as a Prisoners' Dilemma game.

The superpowers each have a choice between two strategies, "Disarm" and "Arm," as shown in Figure 1. The choice of a strategy by both superpowers results in one of the four possible outcomes shown in the *payoff matrix* of Figure 1, which gives all possible outcomes associated with the strategies of each player. An outcome is defined by an ordered pair of numbers (A_i, B_j), where A_i is the payoff to A (row player), B_j the payoff to B (column player).

For player A we assume that A_1 is his best payoff, A_2 next best, A_3 next worst, and A_4 worst; a similar ordering obtains for B. Thus, for example, (A_2, B_2) is a better outcome for both players than (A_3, B_3).

The dilemma in this game is that both players have an unconditionally best, or *dominant*, strategy of Arm: whatever the other player does (Arm or Disarm), each player obtains a higher payoff if he chooses Arm. Thus, a player's "best" strategy choice in Prisoners' Dilemma does not depend on what the other player chooses since a player *always* does better by choosing Arm. Yet, if both players choose Arm, the outcome is (A_3, B_3), which is worse than if both players choose Disarm and thereby obtain (A_2, B_2).

If this is the case, should not both players choose Disarm? The problem here is that (A_2, B_2) is not stable. We say that an outcome is stable, or *in equilibrium*, if, once chosen, neither player can improve his payoff by unilaterally switching to some other strategy.

To show that (A_2, B_2) is not in equilibrium, assume that each player chooses his Disarm strategy associated with this outcome. Then each player has an incentive unilaterally to switch to Arm and thereby obtain his best payoff (A_1 or B_1), inflicting on the other player his worst payoff (B_4 or A_4). This temptation for each player to double-cross the other makes (A_2, B_2) unstable and, we believe, points up the fragility of cooperation (when both players choose Disarm) in the arms race. It is precisely this temptation to double-cross that induces each player to "play it safe" and choose his dominant strategy of Arm, even though the resultant outcome, (A_3, B_3), is the next worst for both players.

The outcome (A_3, B_3), which is circled in Figure 1, is in fact the unique equilibrium outcome in Prisoners'

Dilemma--once chosen, neither player can do better by unilaterally switching to his Disarm strategy. The fact that both players prefer (A_2, B_2) leads us to ask how movement from (A_3, B_3) to (A_2, B_2)--as indicated by the arrow in Figure 1--can be induced, given that (A_2, B_2), once reached, is unstable.

Exercise 1. Construct a payoff matrix in which A_3 and A_4, and B_3 and B_4, are interchanged in Prisoners' Dilemma. Does either player have a dominant strategy in this new game?

Exercise 2. Are there any outcome(s) in equilibrium in this new game?

---

3. INTRODUCING DETECTION PROBABILITIES

Assume that A and B begin the game by both announcing a tit-for-tat policy of *conditional cooperation:* "I'll cooperate (i.e., choose Disarm) if I detect you do; otherwise, I won't." Then, to show their good intentions, assume both players initially cooperate and choose Disarm. This is the *first stage* of the game.[2]

The *second stage* begins when each player makes a second strategy choice, depending on what he detected his opponent did in the first stage. Assume that A can detect with a certain probability the strategy choice of B; and B can likewise detect A's strategy choice.

[2] Other scenarios are, of course, possible, but these moves seem the most plausible to assume if both players are seriously interested in slowing down the arms race. For evidence that this assumption has become reality in the recent period of détente, see Gamson and Modigliani (1971). The rational basis for this assumption in the context of the current arms race is discussed in Section 6.

Specifically, let

- p_A = probability that A can detect B's strategy choice in the first stage;
- p_B = probability that B can detect A's strategy choice in the first stage.

Thus, $0 \leq p_A, p_B \leq 1$.

Consistent with a policy of conditional cooperation, assume that a player chooses Disarm if he detects that his opponent chose Disarm in the first stage; otherwise, he chooses Arm. The question is: does a policy of conditional cooperation benefit the players in the second--and perhaps later--stages of the game?

The *expected payoff* a player derives in the second stage is the sum of the payoffs he obtains from each of four possible outcomes times the probability that each occurs. (The expected payoff in the first stage is A_2 for A and B_2 for B, because by assumption the "cooperative" outcome (A_2, B_2) is chosen with probability 1.) For A, his expected payoff in the second stage will be

$$(1) \quad E(A) = A_2p_Ap_B + A_1(1-p_A)p_B + A_4p_A(1-p_B) + A_3(1-p_A)(1-p_B),$$

assuming A and B make independent strategy choices based solely on their probabilities of detection. Thus, for example, the first term on the right-hand side of (1) says that A and B will correctly detect their mutual choices of Disarm in the first stage with probability p_Ap_B: A will detect B cooperates with probability p_A, and B will detect A cooperates with probability p_B. Now if both players follow a policy of conditional cooperation in the second stage, both will choose Disarm with this probability (p_Ap_B), so A will obtain a payoff of A_2 with probability p_Ap_B (and B will obtain a payoff of B_2 with this probability). The probabilities associated

with the three other payoffs for A in (1) (A_1, A_3, and A_4) can be similarly obtained.

Exercise 3. Write the equation, analogous to (1), for E(B).

Rearranging terms in (1), we obtain

$$(2) \quad E(A) = p_B[A_2p_A + A_1(1-p_A)] + (1-p_B)[A_4p_A + A_3(1-p_A)].$$

Whatever the value of p_A, we know that the first term in brackets on the right-hand side of (2) is always greater than the second term in brackets since $A_2 > A_4$ and $A_1 > A_3$. Therefore, it is in A's interest that p_B be as high as possible (so B will correctly detect cooperation and thereby cooperate himself), and similarly for B with respect to p_A.

Exercise 4. Is the conclusion of the above analysis also true for the players in the game defined in Exercise 1?

This is not a surprising conclusion. Rearranging terms in (1) again, we obtain a more curious result:

$$(3) \quad E(A) = p_A[A_2p_B + A_4(1-p_B)] + (1-p_A)[A_1p_A + A_3(1-p_B)].$$

Now the second term in brackets on the right-hand side of (3) is always greater than the first term in brackets, so it is in A's interest that $(1-p_A)$ be as high as possible, or p_A be as low as possible. This is because A, if he incorrectly detects that B chooses Arm in the first stage and thereby chooses Arm himself in the second stage, obtains a higher expected payoff than if he correctly detects cooperation on the part of B.

But surely B could anticipate this consequence if he knew p_A were low. Hence, B should not mechanically subscribe to a policy of conditional cooperation in the second stage unless he is assured that A can predict with a high probability his cooperative choice in the first stage and thereby respond accordingly. A similar conclusion applies to B. Therefore, it is in the interest of A and B that *both* p_A and p_B be as high as possible.[3]

4. EQUALIZING THE DETECTION PROBABILITIES

How can both players ensure that p_A and p_B are as high as possible? One way, which has been proposed in recent negotiations on a new SALT agreement,[4] is to pool their information so that they both operate from a common (and enlarged) data base. A common data base, presumably, would have the effect of setting the detection probabilities equal to each other. Alternatively, if "national technical means for verification"--in the terminology of currently arms-limitations talks--of both players were equally good, their detection probabilities would also be equal.

To investigate the consequences of equal detection probabilities, assume that $p_A = p_B = p$. The expression for E(A) given by (1) then becomes

$$(4) \quad E(A) = A_2p^2 + (A_1+A_4)(1-p)p + A_3(1-p)^2.$$

[3] For further details, see Brams (1975b). Cf. Howard (1976) for a "general metagames" analysis of Prisoners' Dilemma.

[4] New York Times, April 27, 1977: A7. For an argument that data be collected and verified under international supervision, see Myrdal (1976).

An analogous expression can be obtained for B, but henceforth we shall make only calculations for A since the conclusions we derive apply to B as well.

Without loss of generality, we may assume that the payoffs associated with the best and worst outcomes are one and zero, respectively, i.e., $A_1 = 1$ and $A_4 = 0$. Given this assumption, (4) becomes

$$E(A) = A_2p^2 + (1-p)p + A_3(1-p)^2$$

$$(5) \quad = (A_2+A_3-1)p^2 + (1-2A_3)p + A_3,$$

which is a parabola in p.

Exercise 5. A second-degree curve of the form,

$$Ax^2 + Bx + Cy + D = 0,$$

is a parabola. By making appropriate substitutions, show that (5) is a parabola.

What is of interest is the shape of the parabola in the four regions of the A_2-A_3 coordinate system shown in Figure 2. The shape of the parabola tells us how beneficial a policy of conditional cooperation is as a function of p, assuming (for now) that A_2 and A_3 are fixed.

Since by assumption $0 < A_3 < A_2 < 1$, we need not consider the area on or above the diagonal $A_2 = A_3$. If $(A_2+A_3-1) > 0$, which defines regions I and II, the parabola is concave up; if $(A_2+A_3-1) < 0$, which defines regions III and IV, the parabola is concave down.

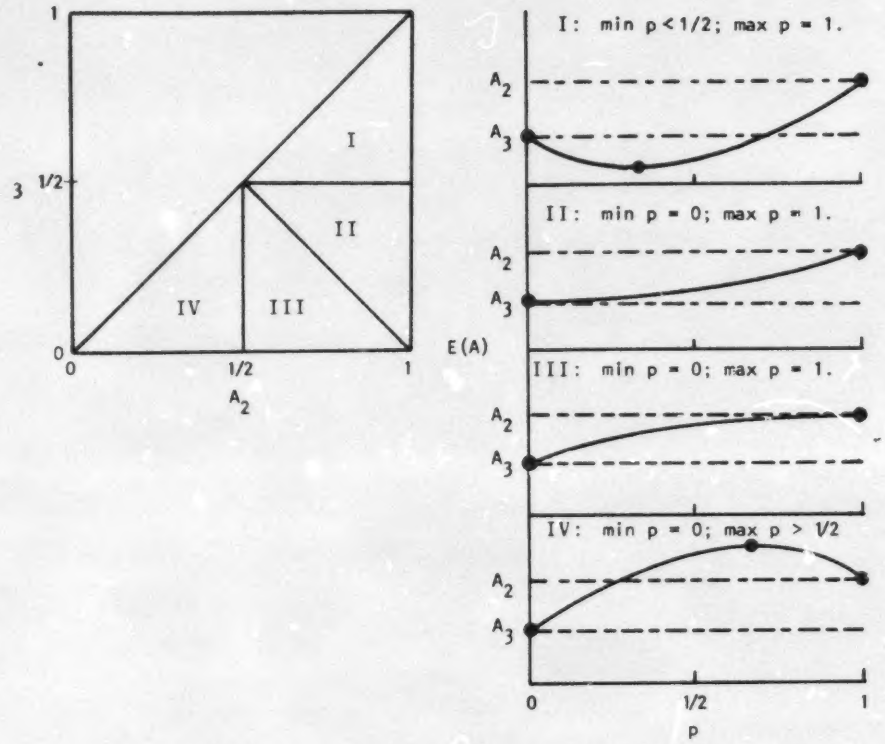

Figure 2. Expected payoffs in four regions.

Exercise 6. Verify that the inequalities given in the previous sentence define the stated regions.

In the interval $0 \leq p \leq 1$, graphs of E(A) (ordinate) as a function of p (abscissa) are shown in Figure 2 for each of the four regions. Note that (i) when $p = 0$, $E(A) = A_3$ and (ii) when $p = 1$, $E(A) = A_2$ in all regions, which can be verified by substituting these values of p into (5).

The vertex of the parabola in all regions is at

$$p = \frac{2A_3 - 1}{2(A_2 + A_3 - 1)}$$

$$(6) \qquad = \frac{(A_3 - 1/2)}{(A_3 - 1/2) + (A_2 - 1/2)}.$$

When substituted into (5), the vertex gives the minimum value of E(A) in regions I and II, the maximum value of E(A) in regions III and IV.

In regions I and II, the denominator of the fraction on the right-hand side of (6) is positive because $(A_2+A_3) > 1$. Clearly, if and only if the numerator is also positive will the minimum of E(A) be at $p > 0$. This occurs in region I, where $A_3 > 1/2$. In region II, where $A_3 < 1/2$, the minimum is at $p < 0$; however, in the interval $0 \leq p \leq 1$, the minimum of E(A) is at the boundary $p = 0$, as shown in Figure 2.

In regions III and IV, both the numerator and denominator of (6) are negative, so the maximum is always at $p > 0$. Rewriting (6),

$$(7) \qquad p = 1 - \frac{(A_2 - 1/2)}{(A_3 - 1/2) + (A_2 - 1/2)},$$

we see that the maximum is at $p < 1$ if and only if the numerator in the fraction on the right-hand side of (7) is negative. This occurs in region IV,[5] where $A_2 < 1/2$. In region III, where $A_2 > 1/2$, the maximum occurs at $p > 1$; however, in the interval $0 \leq p \leq 1$, the maximum of E(A) is at the boundary $p = 1$, as shown in Figure 2.

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[5] Region IV is the only region in which E(A) is not at a maximum when $p = 1$ (in the interval $0 \leq p \leq 1$). This is because $2A_2 < A_1+A_4 = 1$ in this region, so an alternation of the players between their strategies associated with outcomes (A_1, B_4) and (A_4, B_1) yields A a higher expected payoff than does outcome (A_2, B_2). For this reason, Prisoners' Dilemma is sometimes defined so as to preclude payoffs in region IV. See Rapoport and Chammah (1965: 34-35).

Exercise 7 (optional). By setting $\frac{dE(A)}{dp}$ equal to 0, show that E(A) is at an extreme point when

$$p = \frac{2A_3 - 1}{2(A_2 + A_3 - 1)}.$$

Exercise 8 (optional). Find $\frac{d^2E(A)}{dp^2}$ and show under what conditions the extreme point is a maximum and minimum. Does your analysis agree with that in the text?

5. WHEN IS CONDITIONAL COOPERATION RATIONAL?

The graphs of E(A) in Figure 2 show that $E(A) \geq A_3$ for all values of p in regions II, III, and IV. Thus, a policy of conditional cooperation in these regions ensures at least the *security level* of A--the minimum payoff he can ensure for himself, A_3, whatever B does. In fact, this policy will always yield an expected payoff greater than the security level A_3 except when p = 0, which occurs when A always detects the choice of Arm by B, the opposite of what B does.

No such assurance can be offered A if he is in region I. This is the region in which $A_2 > A_3 > 1/2$, i.e, where both the cooperative payoff A_2 and the non-cooperative payoff A_3 are closer to $A_1 = 1$ than $A_4 = 0$. In this case, the loss A suffers from being double-crossed ($A_4 = 0$) is significantly below all his other payoffs.

For this reason, it may be advantageous for A to accept his security level A_3 rather than commit himself to a policy of conditional cooperation. After all, conditional cooperation could result in the payoff $A_4 = 0$, which is much worse than $A_3 > 1/2$ in region I.

In region I, the advantage of A_3 over E(A) is greatest when E(A) is at a minimum, which occurs when $p < 1/2$, as shown in Figure 2. Even for $p \geq 1/2$, however, E(A) may be less than A_3. To determine how high p must be in order that E(A) exceed A_3, we solve

$$(8) \quad E(A) = (A_2 + A_3 - 1)p^2 + (1 - 2A_3)p + A_3 = A_3$$

for p, and get

$$(9) \quad p = 0 \text{ or } p = (2A_3 - 1)/(A_2 + A_3 - 1).$$

We already know $E(A) > A_3$ if $p > 0$ in regions II, III, and IV. In region I, $E(A) > A_3$ if

$$(10) \quad p > \frac{2A_3 - 1}{A_2 + A_3 - 1} = \frac{2(A_3 - 1/2)}{(A_3 - 1/2) + (A_2 - 1/2)}.$$

Algebraic manipulation gives

$$(11) \quad (A_3 - \tfrac{1}{2}) < \frac{p}{2-p}(A_2 - \tfrac{1}{2}).$$

Thus, in region I, a policy of conditional cooperation is better than security level A_3 if the point (A_2, A_3) lies *below* the line which passes through (1/2, 1/2) and has slope $m = p/(2-p)$. For several representative values of p between 0 and 1, these isolines are illustrated in Figure 3 and show that as the detection probability approaches 1, the possibility that conditional cooperation yields less than one's security level vanishes.

Because the slope m of the isolines is convex in p ($\frac{d^2m}{dp^2} > 0$), raising p will make conditional cooperation more advantageous if p is already high. For example, raising p from 3/4 to 1 raises m from 3/5 to 1, or by 2/5, while raising p from 0 to 1/4 only raises m from 0 to 1/7, or by 1/7. Since the base of the traingles (i.e., the abscissa from 1/2 to 1) defining the area in which

$E(A) > A_3$ is the same in each case, and the height is a function of m, the percentage of the total area of the large triangle (at $p = m = 1$) that an increment of 1/4 adds is much greater in the first case (40 percent) than in the second (14 percent). Moreover, since m is always less than 1 except when $p = 1$, raising A_2 (see (11) above) is in general less effective in encouraging conditional cooperation than lowering A_3.

Exercise 9. For the game defined in Exercise 1, find the condition under which $E(A) > A_3$. (Hint: After finding the equation of E(A) analogous to (8), do *not* try to solve for p as in (9). Rather, express the inequality $E(A) > A_3$ as $A_3 > f(p)A_2 + g(p)$. This will facilitate doing Exercise 10 in an A_2-A_3 coordinate system.)

Exercise 10. Illustrate geometrically, as in Figure 3, the meaning of this condition. (Hint: Unlike Figure 3, the A_2 and A_3 coordinates in your graph should range from 0 to 1 since the isolines do not all intersect at point (1/2, 1/2).)

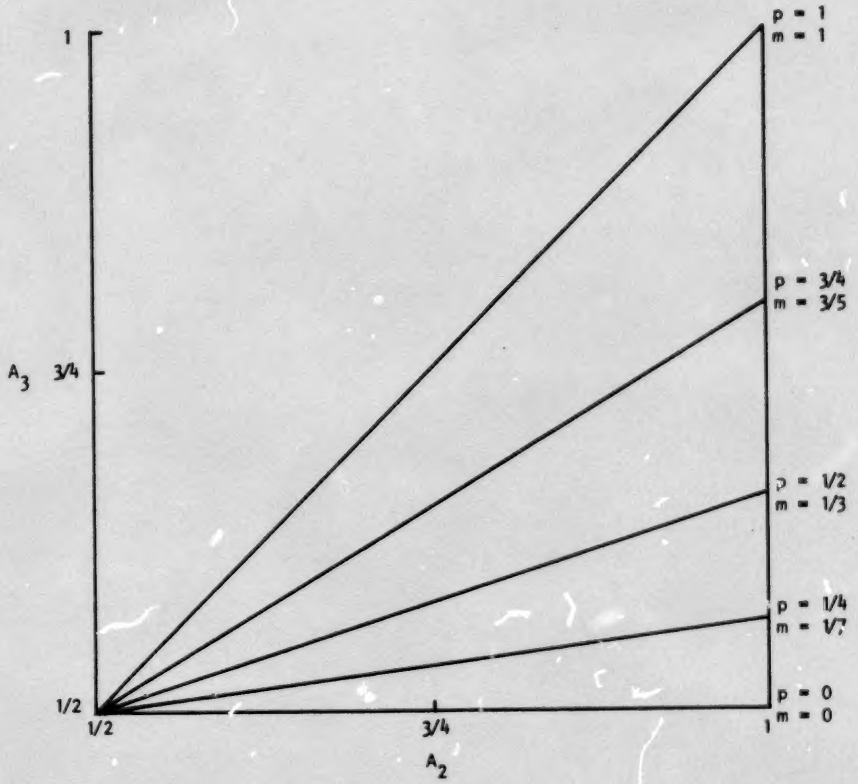

Figure 3. Isolines below which $E(A) > A_3$ in region I.

6. POLICY IMPLICATIONS

We have shown that a policy of conditional cooperation always yields an expected payoff that is at least equal to, and generally exceeds, one's security level in three of the four regions that are feasible for Prisoners' Dilemma when both sides have the same detection probability. In these regions, therefore,

this policy will generally work to the players' mutual advantage, even if the detection probability is low.

Unfortunately, the arms race between the two superpowers probably occurs in region I. Here the consequence of being double-crossed ($A_4 = 0$) is very unsatisfactory compared to accepting one's security level ($A_3 > 1/2$). Yet, our analysis indicates that conditional cooperation even in region I may be beneficial, depending on the detection probability p of both sides. The area in this region where conditional cooperation leads to a higher expected payoff than one's security level increases as (i) p increases; moreover, as (ii) A_2 increases, or (iii) A_3 decreases, the situation is moved rightward and downward, respectively, in Figure 3 toward the area where conditional cooperation is advantageous. It appears that the effects of (i) have already been felt in the limited agreements so far achieved in SALT I and SALT II.

If p continues to increase as technology improves, conditional cooperation should become even more attractive. This is because the slope m increases faster than p when

$$\frac{dm}{dp} > 1,$$

or

$$\frac{2}{(2-p)^2} > 1,$$

$$(12) \quad p > 2 - \sqrt{2} \approx 0.586.$$

Thus, technological improvements that raise p above 0.586 will even more rapidly expand the area in which conditional cooperation is rational for both sides.

We indicated in Section 5 that the effects of (iii) in encouraging conditional cooperation are greater than

the effects of (ii). This means that developments that increase the costs of a continuing arms race (decrease A_3) do more to encourage conditional cooperation than developments that increase the benefits of an arms-control agreement (increase A_2).

Of course, raising the benefits of an agreement and raising the costs of no agreement are two sides of the same coin. But if there is a lesson to be derived from our model, it is that they have unequal trade-offs. Since the multiplier effect is on the cost side of the equation, behavior that raises the costs of an arms race provides the greater incentive for making reciprocal concessions.

Probably the best way to make an arms race more costly is to invest heavily in research and development. This investment increases the probability of technological breakthroughs that create the need for expensive new weapons systems. Paradoxically, perhaps, by making *present* weapons systems more vulnerable to technological breakthroughs, and hence less cost effective, we may better foster a *future* policy conducive to arms-control agreements.

Since the early 1960s, one of the most significant qualitative changes in the nuclear arms race has been the dramatic rise in the detection capabilities of both sides, which has been principally due to the use of reconnaissance satellites.[6] Indeed, President Johnson once stated that space reconnaissance had saved enough in military expenditures to pay for the entire military and space programs.[7]

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[6] Long (1975: 10); Greenwood (1973). For a history of aerial reconnaissance programs since the early 1950s, see York and Greb (1977).

[7] Biddle (1972: 252).

If this detection capability of either side is destroyed or even threatened, then conditional cooperation in region I will once again be rendered unappealing and the prospects of a continuing arms race will be high. On the other hand, if each side's detection capabilities can be ensured or even strengthened--especially through the sharing of data that helps render $p_A = p_B = p$--then further agreements in SALT would appear not only desirable but also rational for both sides.

Just as *stability* in the arms race has depended up to now on the ability of each side to respond to a possible first strike by the other side, *diminution* in the arms race now seems to depend on the ability of each side to detect cooperation on the part of the other side and to respond to it in kind. Unfortunately, "probably nothing the United States does is more closely held than the techniques and performance of its verification machinery."[8] To promote movement toward an arms-control agreement, we believe it is generally in the interest of both the United States and the Soviet Union not only to improve their own detection capabilities but also to abet those of the other superpower.[9]

Naturally, one cannot argue as a blanket prescription that all reconnaissance information about weapons systems should be shared. Information that would greatly increase a country's vulnerability to attack may itself creat instability by making a preemptive strike

[8] Newhouse (1973: 14); security aspects of reconnaissance programs are discussed in Greenwood (1973) and York and Greb (1977).

[9] Cooperation between the superpowers may also work to their advantage with respect to third parties. When the Soviets alerted the United States to possible preparations by South Africa for a nuclear test in August 1977, both countries allegedly worked together to exert politcal pressure that apparently forestalled the test (New York Times, August 28, 1977: 1).

seem more attractive. Thus, the presumed gains in stability both superpowers would buy through a sharing of information that enhances their common detection probability p must be balanced against their increased vulnerability that may be exploited in a first strike that wipes out the ability of a superpower to respond in a putative second stage.

Since we have precluded in our two-stage model noncooperation by either superpower in the first stage, we effectively assume that there is no incentive to strike first. Should this incentive exist, then it would create a fundamental instability that would render our game scenario implausible. However, at this time it seems that both superpowers possess substantial second-strike capabilities, stemming principally from the relative invulnerability of their submarine-launched nuclear missiles. Hence, both superpowers have an incentive not to launch first strikes but instead to find some reasonably safe way to move away from a constant repetition of the burdensome (A_3, B_3) outcome. Our model suggests one way this process may be initiated.

It is important to point out factors that may complicate the rationalistic calculations we have postulated based on the expected-payoff criterion. First, the concept of "expected payoff" assumes that the arms race is not viewed as a one-shot affair but rather as a multi-stage game played out in an uncertain environment. Even viewed in these terms, however, there are many possible scenarios, and we have investigated the consequences of only one. It would be useful to investigate other plausible scenarios--perhaps occurring over more than two stages, possibly with allowance made for the discounting of payoffs in later stages[10]--

[10] For such an approach, see Taylor (1976).

to determine the conditions that make mutual cooperation rational.

Exercise 11. Describe what you consider a plausible scenario and make expected payoff calculations for the players.

Exercise 12. What conclusions to you draw from these calculations?

It would also be useful to investigate how these conditions change when the game being played is different. For example, the game of Chicken, which has been suggested as a model of confrontation situations--like the Cuban missile crisis--in international politics,[11] would be an obvious candidate to which to apply our methodology to determine how sensitive mutual cooperation in this game is to the detection probability p.

Exercise 13. The game defined in Exercise 1 is, in fact, Chicken. On the basis of your calculations for this game in the previous exercises, determine for what values of p the area in which $E(A) > A_3$ is larger for Chicken than Prisoners' Dilemma. What conclusions would you draw from this information?

Another way our analysis might be complicated, and perhaps rendered more realistic, would be to distinguish so-called Type 1 and Type 2 errors. In our model, Type 1 error would refer to incorrectly detecting a violation of a policy of conditional cooperation when in fact there was adherence by the other side, Type 2 error to incorrectly detecting adherence to this policy when in fact there was a violation by the other side, in the second stage of the game scenario. In the

[11] Rapoport (1964); Howard (1971); Brams (1975a).

context of an arms race, there would surely be different reactive strategies associated with each type of error--presumably, Type 2 would cause no change in policy, Type 1 would--and probably different probabilities as well.

Exercise 14. Given our postulated scenario, can there ever be a Type 2 error?

Much work remains to be done to incorporate these and other factors into our present model. We have offered our model primarily to suggest a different way of thinking about arms races--as extended sequences of moves, or scenarios, in multi-stage (versus one-stage) game--that we believe captures interdependencies over time that have not heretofore been modeled. Naturally, we do not mean to imply that national decision makers go exactly through the calculations we set forth or that they are unmoved by nonrational considerations. Rather, we believe that where the stakes are high, as they tend to be in the nuclear arms race, decision makers, at least in a rough way, take account of benefits and costs in the manner postulated in our model.

7. SUMMARY AND CONCLUSION

The arms race between the two superpowers was conceptualized as a Prisoners' Dilemma game, with the additional property that each player can detect initial cooperation or noncooperation on the part of the other player with a specified probability. Consequences of the following scenario were investigated: both players initially cooperate; each player knows the other player's detection probability and follows a policy of conditional cooperation--cooperates if he detects cooperation

on the part of the other player, otherwise does not cooperate.

For the case in which the detection probabilities of the two players are equal, conditional cooperation by both players yielded the following conclusions:

i. Each player's expected payoff as a function of the detection probability is a parabola, which may assume four different forms depending on the payoff each player assigns to the cooperative versus noncooperative outcomes in Prisoners' Dilemma.

ii. The different assignments of payoffs can be represented geometrically by four different regions; in only one of the four regions does conditional cooperation not guarantee a player at least his security level.

iii. Even in this region, as the detection probability approaches one, the possibility that conditional cooperation yields less than one's security level vanishes.

Policy implications of this analysis for SALT were discussed, and a suggestion for the sharing of intelligence data was advanced. It was qualified, however, by noting that enhanced detection capabilities may increase the vulnerability of a country's defenses to a preemptive strike and thereby render a delicate situation more unstable.

Clearly, more attention needs to be paid to the trade-off between the stability induced by better detection capabilities (increasing p) and the instability induced by making a preemptive strike more attractive (rendering our scenario implausible). One thing our model

does tell us is that if there is a choice between making cooperation more attractive (raising A_2), or noncooperation less attractive (lowering A_3), the latter alternative is generally more effective in encouraging conditional cooperation. It perhaps can best be pursued through support of research that renders weapons systems obsolete as rapidly as possible.

We concluded by noting that the methodology of our analysis could be applied to other games (e.g. Chicken) that capture different aspects of conflict in international politics. Scenarios different from the two-stage sequence we postulated earlier might also be explored, with perhaps a discounting factor added in multi-stage games. In this manner, consequences of a variety of games--with different extended sequences of moves--could be investigated that better reflect than one-stage games the dynamic realities of conflict processes.

8. ANSWERS TO EXERCISES

1. No.

		B: Disarm	B: Arm
A	Disarm	(A_2, B_2)	(A_3, B_1)
	Arm	(A_1, B_3)	(A_4, B_4)

2. Yes. Equilibrium outcomes are (A_1, B_3) and (A_3, B_1).

3. $E(B) = B_2p_Ap_B + B_4(1-p_A)p_B + B_1p_A(1-p_B) + B_3(1-p_A)(1-p_B)$.

4. Yes.

5. The appropriate substitutions are:

$x = p$; $y = E(A)$; $A = (A_2+A_3-1)$; $B = (1-2A_3)$; $C = -1$; $D = A_3$.

6. The equation of the line dividing regions II and III is $A_2 + A_3 = 1$, so the region above this line is defined by the inequality $(A_2+A_3) > 1$, the region below this line by the inequality $(A_2+A_3) < 1$.

7. From (5), $\frac{dE(A)}{dp} = 2(A_2+A_3-1)p + (1-2A_3)$. If

$$\frac{dE(A)}{dp} = 0, \quad p = \frac{2A_3-1}{2(A_2+A_3-1)}.$$

8. $\frac{d^2E(A)}{dp^2} = 2(A_2+A_3-1)$.

If $(A_2+A_3-1) < 0$, extreme point is a maximum.
> 0, extreme point is a minimum.

This agrees with the results in the text.

9. From (4), after interchanging A_3 and A_4 and letting $A_1 = 1$ and $A_4 = 0$,

$$E(A) = A_2p^2 + (1+A_3)(1-p)p.$$

Then $E(A) > A_3$ if

$$A_2p^2 + (1+A_3)(1-p)p > A_3$$

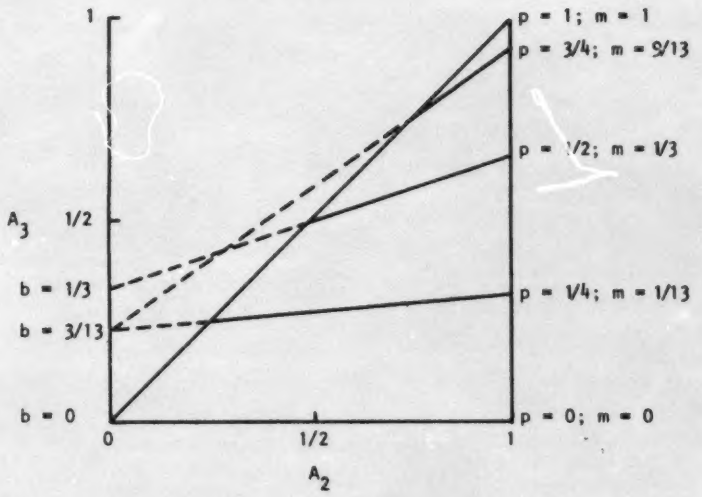
$$A_2p^2 + (1-p)p > A_3(1-p+p^2)$$

$$A_3 < \frac{A_2p^2 + p(1-p)}{(1-p+p^2)}.$$

10. The isolines are straight lines with slope

$$m = \frac{p^2}{(1-p+p^2)} \quad \text{and } A_3 \text{ intercept at } b = \frac{p(1-p)}{(1-p+p^2)}.$$

For representative values of p we have:



11. Not applicable.

12. Not applicable.

13. The area in which $E(A) > A_3$ is

larger for Chicken if $p > 1/2$,
larger for Prisoners' Dilemma if $p < 1/2$,
the same for both games if $p = 1/2$.

Moreover, while for any p in Prisoners' Dilemma, $E(A) \geq A_3$ in regions II, III, and IV of Figure 2, this is true in Chicken only for $p \geq 1/2$. Hence, the policy of conditional cooperation is more advantageous to the players in Prisoners' Dilemma for low p, in Chicken for high p.

14. No.

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STUDENT FORM 1

Request for Help

Return to:
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Student: If you have trouble with a specific part of this unit, please fill out this form and take it to your instructor for assistance. The information you give will help the author to revise the unit.

Your Name_____ Unit No._____

Page_____
- ○ Upper
- ○ Middle
- ○ Lower

OR

Section_____

Paragraph_____

OR

Model Exam
Problem No._____

Text
Problem No._____

Description of Difficulty: (Please be specific)

Instructor: Please indicate your resolution of the difficulty in this box.

○ Corrected errors in materials. List corrections here:

○ Gave student better explanation, example, or procedure than in unit. Give brief outline of your addition here:

○ Assisted student in acquiring general learning and problem-solving skills (not using examples from this unit.)

Instructor's Signature_____

Please use reverse if necessary.

STUDENT FORM 2

Unit Questionnaire

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Name_____ Unit No._____ Date_____

Institution_____ Course No._____

Check the choice for each question that comes closest to your personal opinion.

1. How useful was the amount of detail in the unit?

____Not enough detail to understand the unit
____Unit would have been clearer with more detail
____Appropriate amount of detail
____Unit was occasionally too detailed, but this was not distracting
____Too much detail; I was often distracted

2. How helpful were the problem answers?

____Sample solutions were too brief; I could not do the intermediate steps
____Sufficient information was given to solve the problems
____Sample solutions were too detailed; I didn't need them

3. Except for fulfilling the prerequisites, how much did you use other sources (for example, instructor, friends, or other books) in order to understand the unit?

____A Lot ____Somewhat ____A Little ____Not at all

4. How long was this unit in comparison to the amount of time you generally spend on a lesson (lecture and homework assignment) in a typical math or science course?

Much	Somewhat	About	Somewhat	Much
____Longer	____Longer	____the Same	____Shorter	____Shorter

5. Were any of the following parts of the unit confusing or distracting? (Check as many as apply.)

____Prerequisites
____Statement of skills and concepts (objectives)
____Paragraph headings
____Examples
____Special Assistance Supplement (if present)
____Other, please explain_____

6. Were any of the following parts of the unit particularly helpful? (Check as many as apply.)

____Prerequisites
____Statement of skills and concepts (objectives)
____Examples
____Problems
____Paragraph headings
____Table of Contents
____Special Assistance Supplement (if present)
____Other, please explain_____

Please describe anything in the unit that you did not particularly like.

Please describe anything that you found particularly helpful. (Please use the back of this sheet if you need more space.)